SINGULARITIES IN GENERIC GEOMETRY AND ITS APPLICATIONS: VALENCIA V
Talks and abstracts

1 PLENARY TALKS

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Cobordism of maps between singular spaces

This is a lecture on a join work with Alice Libardi and Eliris Rizziolli from UNESP, Rio Claro and Marcelo Saia, USP, São Carlos, Brasil. In order to extend the theory of cobordism of maps between smooth manifolds to maps between singular spaces, one meet various obstacles. On the one hand, one has to consider suitable maps and spaces, namely so-called normally non-singular maps and Witt spaces. On the other hand, the Stiefel-Whitney classes cannot be generalized in ordinary homology, one has to consider intersection homology in order to be able to multiply such classes in the singular case. In such a situation, the authors prove some results such as null-cobordism implies the vanishing of Stiefel-Whitney numbers. In the lecture motivations and basic definitions and properties for singular spaces will be recalled.

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On the theory of nonlocal normal forms of mixed type PDE's on the plane

The local classification of generic linear second order mixed type PDE's on the plane was completed in [1], [2].

Later on some analogous results were obtained in a similar classification for typical families of such equations [3], [4], [5], [6], [7]. However, the complete smooth classification, even for typical one-parameter families, has not yet been obtained.

We discuss these results and open problems in the theory of normal forms of typical families of second-order linear partial equations on the plane.

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G-structures and singularities of map-germs

We consider equivalence relations among smooth section-germs of vector bundles with respect to structure groups. One of these equivalence relations is the G-equivalence among smooth map germs introduced by Tougeron [5]. Alltough Gervais [1, 2, 3] investigated some properties of G-equivalence, there are no proper examples in their contexts. We give several interesting applications including quantum chemistry and spintronics [4], which are expected to have an application to the theory of topological insulators and so on. We emphasize that we can open a door for new important applications of singularity theory. Moreover we define other equivalence relations with respect to structure groups of vector bundles. Since a vector space can be identified with the tangent space of itself, we can consider the structure group of the vector bundle as a G-structure of the fiber (the vector space). Then we discover that there are a lot of previous applications or new applications of the theory of singularities as examples of these equivalence relations. Some of these equivalence relations are geometric subgroups of $\mathcal{A}$ or $\mathcal{K}$ in the sense of Damon. However, some of them are not.

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Differential forms on varieties, symplectic and affine invariants

We study germs of differential forms over singular varieties. Their residues, i.e. algebraic restrictions with vanishing geometric restrictions over plane curves, hypersurfaces and Lagrangian varietes as well as we show this is a powerful metod to investigate symplectic invariants of singularities. Joint work with G. Ishikawa, W. Domitrz, P. Giblin and Z. Jelonek is reported.

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Intersections of ellipsoids and intersections of hyperboloids related to quadratic singularities

(joint work with Vinicio Gómez Gutiérrez)

In the study of singularities of homogeneous quadratic maps $f : \mathbb{R}^n \to \mathbb{R}^m$ for $n \geq m$, the zero set V of f plays an important role. Intersections of concentric ellipsoids appear as the link of $V$and intersections of concentric hyperboloids as deformations of V. In previous versions of this work these intersections were vaguely described as "intersections of quadrics". Here we will present this work under this new approach as well as some recent results. We will begin by characterizing the topological type of those varieties which are transverse intersections of general ellipsoids in $\mathbb{R}^n$. Then we will describe the relations between the different kinds of intersections. For $m \leq 2$ we will describe the topology of all the generic links and (almost) all deformations. For $m > 2$ the difficulties for describing all of them look unsurmountable but we will nevertheless describe the topology of large families of them. Finally, we will mention some potential applications to the Generic Geometry of codimension 2 submanifolds of $\mathbb{R}^n$.

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Inflection Points on Hyperbolic tori of $\mathbb{S}^3$

Families of hyperbolic tori in $\mathbb{S}^3$ (the asymptotic lines are globally defined) without double inflection points is provided. More precisely, a small deformation of the Clifford torus parametrized by asymptotic lines is analyzed and it is described the set of inflections of the two families of asymptotic lines $\mathcal{A}_1$ and $\mathcal{A}_2$. Denote by F_i the set of inflections of the asymptotic lines of the associated asymptotic foliation $\mathcal{A}_2 i$, also called flecnodal set. The intersection $F_1 \cap F_2$ is called the set of double inflection. It is shown that by an appropriated deformation of the Clifford torus the set $F_1 \cap F_2$ can be empty for the deformed surface. This gives a partial negative answer to a problem formulated by S. Tabachnikov and V. Ovskienko [Hyperbolic Carathéodory Conjecture, Proc. of the Steklo Inst. of Math. **258** (2007), p. 178-193] in the context of spherical surfaces.

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On the geometry of the cuspidal cross-cap

We study the geometry of the cuspidal cross-cap M in $\mathbb{R}^3$ obtained by folding generically a cuspidal edge. We study geometrical invariants associated to M and show that they determine it up to order 5. We then study the flat geometry (contact with planes) of a generic cuspidal cross-cap by classifying submersions which preserve it and relate the singularities of the resulting height functions with the geometric invariants. Joint work with K. Saji.

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Geometry of Severi Strata

The classical subject of plane curve singularities contains many mysteries. In the talk I will speak on the strata in the versal deformation of a curve singularity $(C, 0)$ over which the curve contains a prescribed number k of ordinary double points. For $k = 1$ this is just the discriminant, whereas for $k = \delta(C, 0)$ it is known as the δ-constant stratum. We will report on conjectures and theorems on these strata.

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Families of spherical surfaces and harmonic maps

(joint work with David Brander)

We shall present results on singularities of constant positive Gaussian curvature surfaces and on their bifurcations in generic 1-parameter families of such surfaces. These surfaces are related to harmonic maps. We shall also present some results on the local singularities of these maps.

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Support function, big front singularities and duality

The so-called "support function" of a given closed convex plane curve enables to describe the equidistant curves and their singularities. We show that the graph of the support function contains all local and global geometric information of the initial curve, of its equidistants and of its evolute (caustic). To any plane curve (without convexity restrictions) corresponds a curve on the unit cylinder (the graph of a "multivalued support function") and vice-versa. We define the "support map", which sends any plane curve to a curve on the unit cylinder and establish the correspondence between Euclidean differential geometry of plane curves and projective differential geometry of curves on the unit cylinder.

We geometrically construct the natural isomorphism between the front (in space-time) formed by the union of equidistants of a plane curve and the dual surface of its corresponding curve on the cylinder (the subvariety formed by the planes of R^3 which are tangent to this space curve). Our results hold in Euclidean spaces of higher dimensions for submanifolds of any dimension.

A corollary of our construction is the following:

Theorem. For any class of singularities X (for example, A, D, E) the set of singularities of type X of the evolute of a smooth submanifold M of R^n is isomorphic to the set of singularities of type X in the front formed by the hyperplanes of R^{n+1} which are tangent to the image of M by the support map (in the unit cylinder $C_n \subset R^{n+1}$) by the support map.

The results are explained by the natural contactomorphism between $J^1(S^{n-1}, R)$ and ST^*R^n and their relations with $J^1(R^n, R)$ and T^*R^n where a Legendrian manifold of $J^1(R^n, R)$ is also a Lagrangian manifold of T^*R^n.

The talk will be elementary and with many pictures.

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Topology of Determinantal Singularities

A Determinantal Singularity X is described as the vanishing locus of certain minors of a matrix A. The deformations of X coming from perturbations of A are particulary well behaved and if X is "isolated" there is a unique determinantal Milnor fiber associated to it. It turns out that the topological invariants of these Milnor fibers differ significantly from those arising from isolated complete intersection singularities. Namely, there are vanishing cycles which do not depend on the specific entries of A but arise from the determinantal structure only. They also contribute to the homology below the middle degree.

2 Talks.

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About the geometry of corank 1 surfaces in $\mathbb{R}^4$

This talk is divided in two parts. In the first one, we present a more general study about the geometry of surfaces in $\mathbb{R}^4$ with corank 1 singularities. At the singular point we define the curvature parabola using the first and second fundamental forms of the surface. R. Mendes and J.J. Nuño-Ballesteros give in [5] a partition in four orbits of all corank 1 map germs $f : (\mathbb{R}^2, 0) \to (\mathbb{R}^4, 0)$ (see [2]) according to their 2-jets under the action of $\mathcal{A}^2$. We show that the curvature parabola distinguishes the four types of corank 1 singularities only by looking at the type of degeneracy of the parabola. We also show that two corank 1 2-jets $(\mathbb{R}^2, 0) \to (\mathbb{R}^4, 0)$ are equivalent under the action of the subgroup $\mathcal{R}^2 \times \mathcal{O}(4)$ iff there exists an isometry between the normal planes preserving the respective curvature parabola. Therefore, the curvature parabola contains all the local second order geometrical information of the surface. Inspired by [4] and [3] the definitions and some results about asymptotic directions and umbilic curvature are given. The second part is dedicated to the classification of submersions $(\mathbb{R}^4, 0) \to (\mathbb{R}, 0)$ up to change of coordinates in the source that preserve the model surface `X` given by $(x,y) \mapsto (x, xy, y^2, y^3)$. Those change of coordinates form a geometric subgroup $\mathcal{R}(\mathtt{X})$ of the Mather group $\mathcal{R}$. This method is used in [1] and [6], where the authors classify submersions $(\mathbb{R}^3, 0) \to (\mathbb{R}, 0)$ that preserves the crosscap and the cuspidal edge, respectively. Using this classification we study the generic geometry related to the contact between X and hyperplanes in $\mathbb{R}^4$. Such contact is measured by the singularities of the height function of X. The results are part of my PhD thesis supervised by M. A. S. Ruas and it is a joint work with M. A. S. Ruas and R. Oset Sinha.

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The special closure of polynomial maps and global non-degeneracy

Let $F : \mathbb{C}^n \to \mathbb{C}^n$ be a polynomial map such that $F^{-1}(0)$ is finite. We analyze the connections between the multiplicity of F, the Newton polyhedron of F and the set of special monomials with respect to F, which is a notion motivated by the integral closure of ideals in the ring of analytic function germs $(\mathbb{C}^n, 0) \to \mathbb{C}$. In particular, we characterize the polynomial maps whose set of special monomials is maximal. We also derive results about the lower estimation of the Łojasiewicz exponent of F at infinity, the injectivity of F and the computation of global Milnor numbers. This is part of a joint work with Jorge A.C. Huarcaya.

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Duality and horocyclic evolutes of fronts in hyperbolic space

(joint work with Shyuichi Izumiya and Masamoto Takahashi)

We investigate geometric properties of a special kind of evolutes, so called horocyclic evolutes, of smooth curves in hyperbolic space. To do that, we first review the basic notions of fronts in hyperbolic 2-space which developed by the first and the third authors by using basic Legendrian duality theorem developed by the second author. Moreover, two kinds of horocyclic evolutes are defined and the relationship between these two different evolutes are studied. As results, they are Legendrian duality to each other and both can be viewed as wavefronts from the Legendrian singularity theory viewpoint.

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Quadraticpoints and affine umbilics of surfaces in 3-space

Quadratic and umbilic are important points in affine differential geometry of surfaces in 3-space. Around such points, there are some significant directions: Principal directions close to an affine umbilic and directions with vanishing cubic form close to a quadratic point. In this talk, we shall discuss the local behavior of isolated quadratic and affine umbilic points of convex and non-convex surfaces.

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Binary Differential equations at parabolic and umbilical points for 2-parameter families of surfaces

(Jorge Luiz Deolindo Silva, Yutaro Kabata, and Toru Ohomoto)

We determine local topological types of binary differential equations of asymptotic curves at parabolic and flat umbilical points for generic 2-parameter families of surfaces in $\mathbb{P}^3$ by comparing our projective classification of Monge forms and classification of general BDE obtained by Tari and Oliver. In particular, generic bifurcations of the parabolic curve are classified and the flecnodal curve is also examined by direct computations.

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On local invariants of singular symplectic forms

We find a complete set of local invariants of singular symplectic forms with the structurally stable Martinet hypersurface on a $2n$-dimensional manifold. In the $\mathbb{C}$-analytic category this set consists of the Martinet hypersurface Σ_2, the restriction of the singular symplectic form ω to $T\Sigma_2$ and the kernel of ω^{n-1} at the point $p \in \Sigma_2$. In the $\mathbb{R}$-analytic and smooth categories this set contains one more invariant: the canonical orientation of Σ_2. We find the conditions to determine the kernel of ω^{n-1} at p by the other invariants. In dimension 4 we find sufficient conditions to determine the equivalence class of a singular symplectic form-germ with the structurally smooth Martinet hypersurface by the Martinet hypersurface and the restriction of the singular symplectic form to it. We also study the singular symplectic forms with singular Martinet hypersurfaces. We prove that the equivalence class of such singular symplectic form-germ is determined by the Martinet hypersurface, the canonical orientation of its regular part and the restriction of the singular symplectic form to its regular part if the Martinet hypersurface is a quasi-homogeneous hypersurface with an isolated singularity.

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Direction Fields Defined by Line Bitangencies

Given two generic immersions of surfaces in three-dimensional Euclidean space, the set of line bitangencies make a two-dimensional set. The directions associated to the line bitangencies give a multi-valued direction field on each surface. In this talk, we look at the local structure of this direction field, especially near the critical set of the line bitangencies. We compare the behavior to that of binary differential equations, we see how the local structure changes in a generic one-dimensional family of immersions, and we use the local structure to prove global results about line bitangencies. Finally, we find necessary and sufficient conditions for a locally defined direction field to be part of a line bitangency direction field.

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Generic linear perturbations

In his celebrated paper "Generic projections", John Mather has shown that almost all linear projections from a submanifold of a vector space into a subspace yield a stable mapping in the nice dimensions. In this talk, an improvement of the Mather result is given. Namely, almost all linear perturbations of a smooth mapping from a submanifold of $\mathbb{R}^m$ into $\mathbb{R}^\ell$ yield a stable mapping in the nice dimensions.

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Duality-singularity of indefinite metrics and contact structures

An immersed surface in a Lorentzian manifold is called "null" if the induced metric is degenerate everywhere on the surface. Null surfaces play important role in Lorentzian geometry, however they possess singularities naturally. The study of null surfaces in a three dimensional Lorentzian manifoldleads us to find connections with the contact theory of third order ordinary equations and Legendre path geometry. In fact, it is well known the connection with the study of differential equations and conformal geometry.

We study the singularities of null surfaces in the natural framework of diality involved with contact geometry and pseudo-product Engel structure which is naturally associated with three dimensional Lorentzian geometry.

To perform the exact classification of singularities appearing in null surfaces, we introduce the notion of "null frontals" in a natural way. Then we present several classification results of singularities which arise in null frontals up to local diffeomorphisms.

The classification is achieved by using the fact that null frontals are obtained as tangent surfaces to directed null curves, namely, ruled surfaces by tangent null geodesics along null curves in the Lorentzian manifold, as well as associated varieties to Legendre curves in the space of null geodesics.

Some generalizations to higher dimensional cases will be discusssed. This is a talk based on a series of jointworks with Y. Machida and M. Takahashi.

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Classification of jets of ruled surfaces in 3-space

We present normal forms of jets of ruled surfaces in 3-space with codimension ≤ 3 via projective transformations. This generalizes classical results with the case of codimension 0 due to Wilczynski and Darboux, and leads to new interests in applications.

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Equivariant indices of 1-forms on varieties

(S.M. Gusein-Zade, F.I. Mamedova)

For a G-invariant holomorphic 1-form with an isolated singular point on a germ of a complex-analytic G-variety with an isolated singular point (G is a finite group) one has notions of the equivariant homological index and of the (reduced) equivariant radial index as elements of the ring of complex representations of the group. We show that on a germ of a smooth complex-analytic G-variety these indices coincide. This permits to consider the difference between them as a version of the equivariant Milnor number of a germ a G-variety with an isolated singular point.

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Extrinsic Geometry of Surfaces in $\mathbb{R}^5$

(joint work with Pierre Bayard and Federico Sánchez-Bringas)

We study the extrinsic geometry of a surface in $\mathbb{R}^5$ in relation to contact theory. We first completely determine the numerical invariants of the second fundamental form and describe the corresponding curvature ellipse. We then introduce and study a new quadratic map closely related to the degenerate directions of the surface and characterize inflection and umbilic points regarding the invariants. An intrinsic equation of the asymptotic lines is provided as a result of this approach. We analyze the Gauss map form the surface into the Grassmannian of 2-planes in $\mathbb{R}^5$, and characterize the points where this map is not regular in terms of contact in the Grassmannian. Finally, we give a simple condition which guarantees the existence of an isometric reduction of codimension of the surface into $\mathbb{R}^4$, and show an example that illustrates such a reduction of codimension.

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Stability of multiplicity and degree with respect to sublevel sets

Let S be an unbounded subset of R^n. Consider a polynomial f. Let $\deg_S f$ be the smallest degree of a polynomial h such that $f < h$ on S. We call such a number the degree of f relative to S. Analogously, one can define a multiplicity at 0 relative to a set S such that 0 lies in its closure.

Consider a real polynomial mapping $(g_1, \ldots, g_k) : R^n \to R^k$ and its sublevel set S_c, where $c \in R^k$, given by inequalities $g_1 < c_1, \ldots, g_k < c_k$. We show that there exists a semialgebraic set $V_g \subset R^k$ of positive codimension such that if c, C are contained in the same connected component of $R^k \setminus V_g$, then the relative degrees coincide i.e. $\deg_{S_c} \equiv \deg_{S_C}$. Analogous property is true for the relative multiplicity. To prove this, we will construct an appropriate compactification of R^n via resolution of singularities.

We will discuss the relation of V_g with bifurcation values at infinity of g, the moment problems and Positivstellensaetze. This is joint work with V. Grandjean.

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Asymptotic directions of spacelike surfaces in de Sitter 5-space

(work in progress with M. Kasedo and M.A.S. Ruas)

De Sitter space is known as a Lorentz space with a positive constant curvature in the Minkowski space. A Surface in Minkowski space with a Riemannian metric is called a spacelike surface. The aim of this work is to investigate geometric properties of spacelike surfaces in the de Sitter space, specially properties of the asymptotic directions of such surfaces. We use techniques from singularity theory.

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Bi-center problem for some classes of Z2-equivariant systems

In a real planar analytic differential system a singular point with pure imaginary eigenvalues of the matrix of the linear approximation can be either a focus or a center. The problem to distinguish between a center or a focus is called the center problem. If the singular point is a center the next arising problem is to determine whether the center is isochronous, that is, whether all solutions near the singular point have the same period. Although the center problem have been studied during more than hundred years by many authors it is unresolved even for planar systems with cubic nonlinearities. The existence of two simultaneous centers in planar differential systems was investigated only for very few particular families of systems. In this talk we shall discuss the simultaneous existence of two centers and their isochronicity for two families of planar $\mathbb{Z}_2$-equivariant differential systems. Liu and Li (Acta Math. Sin, 2011) presented the necessary and sufficient conditions for a $\mathbb{Z}_2$-equivariant cubic system have a bi-center at the points (1,0) and (-1,0). In this talk we shall discuss the necessary and sufficient conditions for the existence of an isochronous bi-center for such system. Next, we extend the study to quintic systems giving conditions for the existence of a bi-center and studing its isochronicity for a planar $\mathbb{Z}_2$-equivariant quintic system having two weak foci or centers. From this study we also have two examples of non-Hamiltonian cubic systems with three isochronous centers. This is a joint work with Wilker Fernandes (ICMC-USP) and Valery Romanovski (CAMPT-Eslovenia).

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Curves in the Minkowski plane and their geometry

The equivalence relation by the group $\mathcal{A}$ is very important for studying singularities of map-germs, in particular, when we study the deformations of singularities of plane curves. However, if we are interested in the geometry of curves, then this relation is not very appropriate, since diffeomorphisms do not preserve their geometry. Hence arose the need to define deformations which preserves the geometry of curves. In [SALARINOGHABI, M.; TARI, F. Flat and round singularity theory of plane curves. Preprint, 2015.], such deformations were called FRS-deformations, from which it was possible to obtain a way to study the bifurcations of a curve in the Euclidian plane taking into account its geometry (inflections, vertices and singularities).

Our goal is to consider the above questions in the case of curves in the Minkowski plane. We note that the geometric consequences of the contact of a curve with pseudo-circles in the Minkowski plane are different of the contact of a curve with a circle in the Euclidian plane, as, for example, the behavior of the caustic.

In this work we define the FRLS-equivalence of curves in the Minkowski plane and we analyzed the deformations of curves and the bifurcation of its caustics in the Minkowski plane around specific points, such as vertices, inflections and lightlike points.

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Cusps of curves in Euclidean 3-space

(joint work with Tongchang Liu and Cuilian Zhang)

In this talk, we define the $(n, m)-$cusp curve and give the local theory of $(n, m)-$cusp curves. Moreover, we define the modified Frenet-Serre type frame and study the behavior of curvature functions at a $(n, m)-$cusp. As an application, we introduce the notion of a $(n, m)-$cusp helix which is a generalization of a classical general helix.

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Reflection Maps

In this talk we introduce the family of reflection maps, a generalization of the fold and double-fold families. Given a reflection group G acting on $\mathbb{C}^p$, a reflection map $f\colon X \to \mathbb{C}^p$ is the composition of an embedding $h\colon X \hookrightarrow \mathbb{C}^p$ with the orbit map $\omega\colon \mathbb{C}^p \to \mathbb{C}^p$ that maps the G-orbit of a point to a single point. We give obstructions to stability and cA-finiteness of reflection maps and show, in the non-obstructed cases, infinite families of cA-finite map-germs of any corank.

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Singularities of Equidistants and Super Caustics in 3-Space

I shall discuss the local geometry of a generic 1-parameter family of smooth surfaces in 3-space for which one member of the family has parallel tangent planes at two parabolic points with the additional property that their unique asymptotic directions are also parallel. I shall consider the equidistants of this family, that is the loci of points at a fixed ratio along chords joining points with parallel tangents, as a 2-parameter family depending on the value of the fixed ratio and on the parameter in the family of curves. This scenario gives rise to a so-called super caustic and is analogous to our previous works concerning planar super caustics. In the plane case the "gull" singularity arose and in the present 3-dimensional work there appears a 3-dimensional equivalent gull singularity as well as some new classes. This is joint work with Peter Giblin.

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Explicit examples of surfaces in $\mathbb{R}^4$ with constant Jordan angles

In a recent paper by P. Bayard *et al.*, *Surfaces in $\mathbb{R}^4$ with constant principal angles with respect to a plane* (CJA), one can found a non constructive proof of the existence of an immersion with the CJA property for a given pair of angles. In this contribution, an alternative approach is given which allow us to build explicit examples of some CJA immersions for any pair of angles. In addition, we study the uniqueness of these examples for some given conditions.